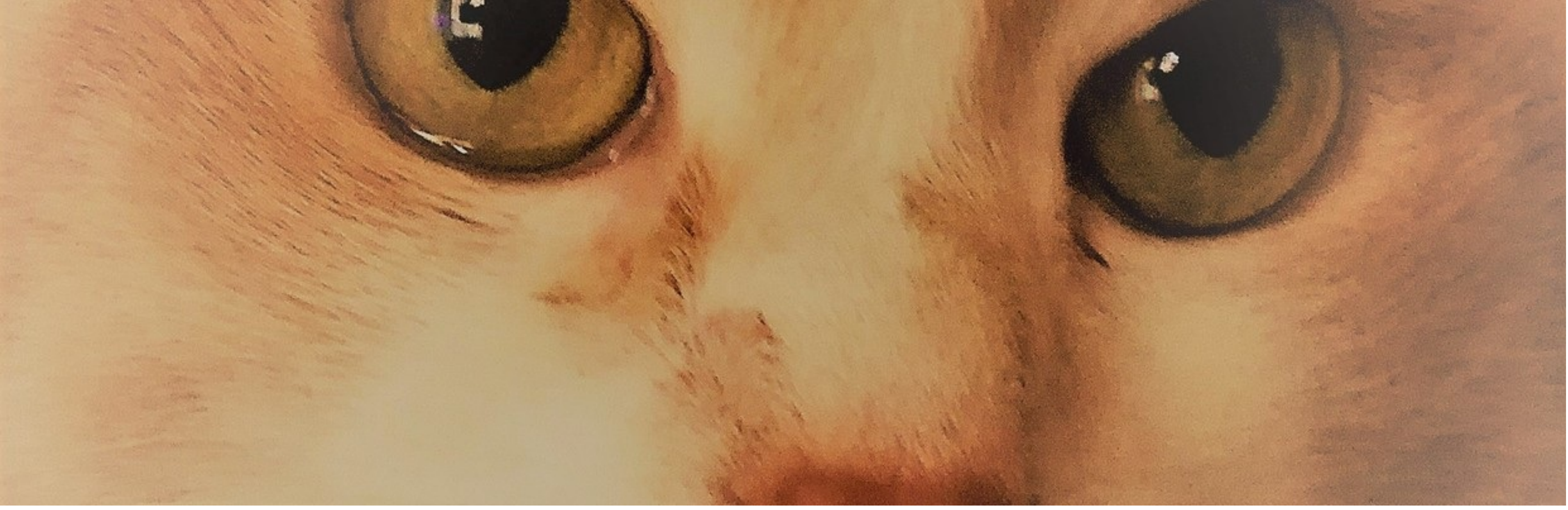

The Evolution of Harmonic Progression Analysis: Ultimate CAT

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The Evolution of Harmonic Progression Analysis: Ultimate CAT

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Abstract

In this article we introduce an improved version of *CAT* (Cataldo Advanced Transformations). The purpose of *CAT* fundamentally lies in providing musicians with a reliable instrument that may effectively assist them in carrying out chord progressions analysis. The correct application of *CAT* allows to convert any harmonic progression, net of some very rare exceptions, into a mere sequence of plagal and perfect cadences. Amongst other features, the improved version of *CAT*, unlike the previous one, takes also into account Modal Interchange and Tonicization.

Keywords

CAT, Cataldo Advanced Transformations, Chord Progression Analysis, Harmonic Substitutions, Jazz.

Introduction

Net of a single exception (a routine named "Structure Reduction"), the whole method is based on the mere application of a series of (inverse) harmonic transformations [1] [2]. The above-mentioned transformations, herein introduced in a considerably improved version, turn out to be nothing but inverse chord substitutions characterized by specific conditions and restrictions. As elsewhere underlined [7] [8], the method arises from the analysis of a considerable number of chord progressions, devoting particular (although not exclusive) attention to traditional jazz compositions. A significant improvement of *CAT* has been achieved by conducting an extremely thorough analysis of a huge amount of *LEGO Bricks* (public domain harmonic patterns) [5] [6]. Unlike the previous version [3] [4], the one herein introduced has no limitation concerning the key (any song written in both major and minor key can be analysed), exploits a more rigorous definition of "Similitude", and takes into consideration "Modal Interchange" (comparing 35 scales/modes) and "Tonicization" (actually not to be regarded as a real substitution, but rather as a "Harmonic Enrichment") [7] [8]. The time signature must always be imagined as being equal to *4/4*. For example, even if we deal with a *3/4*, we have to consider four pulses per measure (four beats per bar): each beat, in this case, will be characterized by a duration equivalent to a dotted quaver.

1. KEY Selection and Writing of the SCALE and HARMONIZATION VECTORS

If we denote with *X* a generic note belonging to the *Chromatic* scale, and with *t* a whole tone interval, we can represent the *Ionian* and the *Aeolian* scales as vectors [9] [10], denoted by s^{Ion} and s^{Aeo} respectively [11] [12]. Similarly, we can consider two vectors, denoted by h^{Ion} and h^{Aeo}, whose components are the seventh chords that arise from the harmonization of the above-mentioned scales.

(X) MAJOR Key

$$\boldsymbol{s}^{Ion}(X) = \begin{bmatrix} X \\ X+t \\ X+2t \\ X+\frac{5}{2}t \\ X+\frac{7}{2}t \\ X+\frac{9}{2}t \\ X+\frac{11}{2}t \end{bmatrix} \qquad \boldsymbol{h}^{Ion}(X) = \begin{bmatrix} Xmaj7 \\ \langle X+t\rangle m7 \\ \langle X+2t\rangle m7 \\ \langle X+\frac{5}{2}t\rangle maj7 \\ \langle X+\frac{7}{2}t\rangle 7 \\ \langle X+\frac{9}{2}t\rangle m7 \\ \langle X+\frac{11}{2}t\rangle m7b5 \end{bmatrix}$$

(X) MINOR Key

$$\boldsymbol{s}^{Aeo}(X) = \begin{bmatrix} X \\ X+t \\ X+\frac{3}{2}t \\ X+\frac{5}{2}t \\ X+\frac{7}{2}t \\ X+4t \\ X+5t \end{bmatrix} \qquad \boldsymbol{h}^{Aeo}(X) = \begin{bmatrix} Xm7 \\ \langle X+t\rangle m7b5 \\ \langle X+\frac{3}{2}t\rangle maj7 \\ \langle X+\frac{5}{2}t\rangle m7 \\ \langle X+\frac{7}{2}t\rangle m7 \\ \langle X+4t\rangle maj7 \\ \langle X+5t\rangle 7 \end{bmatrix}$$

2. STRUCTURE REDUCTION

Structure Reduction – Without "Structure Reduction" a correct application of *CAT* (*Cataldo Advanced Transformations*) is de facto impossible. Very simply, the number of bars and the duration of the chords are iteratively halved. The procedure is stopped the moment in which even a single chord characterized by a duration equal to a beat appears. "Structure Reduction" is applied every time it is possible, so as to obtain the highest simplification level. [3] [4]

Very concisely, we have:

$$n_{max} = current\ number\ of\ chords$$

$$k_{max} = current\ number\ of\ bars$$

$$a_n = n - th\ chord$$

$$T(a_n) = current\ duration\ of\ the\ n - th\ chord$$

$$\forall n \in (1, n_{max}) \quad T(a_n) > 1beat\ \ and\ \ \frac{k_{max}}{2} \in N \implies T(a_n) \xrightarrow{red.} \frac{T(a_n)}{2}\ \ and\ \ k_{max} \xrightarrow{red.} \frac{k_{max}}{2}$$

3. Inverse Substitutions (Modal Interchange and Similitude) involving AUGMENTED MAJOR SEVENTH CHORDS

Modal Interchange – Two chords that arise from the harmonization of two different scales characterized by the same tonic (generic parallel keys) are interchangeable if they are placed in the same position (if they represent the same harmonic degree). [7] [8]

$X = C$	h^1	h^2	h^3	h^4	h^5	h^6	h^7
Ionian	Cmaj7	Dm7	Em7	Fmaj7	G7	Am7	Bm7b5
Dorian	Cm7	Dm7	E^{b}maj7	F7	Gm7	Am7b5	B^{b}maj7
Phrygian	Cm7	D^{b}maj7	E^{b}7	Fm7	Gm7b5	A^{b}maj7	B^{b}m7
Lydian	Cmaj7	D7	Em7	F$^\#$m7b5	Gmaj7	Am7	Bm7
Mixolydian	C7	Dm7	Em7b5	Fmaj7	Gm7	Am7	B^{b}maj7
Aeolian	Cm7	Dm7b5	E^{b}maj7	Fm7	Gm7	A^{b}maj7	B^{b}7
Locrian	Cm7b5	D^{b}maj7	E^{b}m7	Fm7	G^{b}maj7	A^{b}7	B^{b}m7
Ipoionian	Cm$^\Delta$	Dm7	E^{b}maj7#5	F7	G7	Am7b5	Bm7b5
Dorian b2	Cm7	D^{b}maj7#5	E^{b}7	F7	Gm7b5	Am7b5	B^{b}m$^\Delta$
Lydian Augmented	Cmaj7#5	D7	E7	F$^\#$m7b5	G$^\#$m7b5	Am$^\Delta$	Bm7
Lydian Dominant	C7	D7	Em7b5	F$^\#$m7b5	Gm$^\Delta$	Am7	B^{b}maj7#5
Mixolydian b6	C7	Dm7b5	Em7b5	Fm$^\Delta$	Gm7	A^{b}maj7#5	B^{b}7
Locrian #2	Cm7b5	Dm7b5	E^{b}m$^\Delta$	Fm7	G^{b}maj7#5	A^{b}7	B^{b}7
Superlocrian	Cm7b5	D^{b}m$^\Delta$	E^{b}m7	F^{b}maj7#5	G^{b}7	A^{b}7	B^{b}m7b5
Harmonic Minor	Cm$^\Delta$	Dm7b5	E^{b}maj7#5	Fm7	G7	A^{b}maj7	Bdim7
Locrian #6	Cm7b5	D^{b}maj7#5	E^{b}m7	F7	G^{b}maj7	Adim7	B^{b}m$^\Delta$
Ionian Augmented	Cmaj7#5	Dm7	E7	Fmaj7	G$^\#$dim7	Am$^\Delta$	Bm7b5
Romanian	Cm7	D7	E^{b}maj7	F$^\#$dim7	Gm$^\Delta$	Am7b5	B^{b}maj7#5
Phrygian Dominant	C7	D^{b}maj7	Edim7	Fm$^\Delta$	Gm7b5	A^{b}maj7#5	B^{b}m7
Lydian #2	Cmaj7	D$^\#$dim7	Em$^\Delta$	F$^\#$m7b5	Gmaj7#5	Am7	B7
Ultralocrian	Cdim7	D^{b}m$^\Delta$	E^{b}m7b5	F^{b}maj7#5	G^{b}m7	A^{b}7	B^{bb}maj7
Harmonic Major	Cmaj7	Dm7b5	Em7	Fm7	G7	A^{b}maj7#5	Bdim7
Dorian b5	Cm7b5	Dm7	E^{b}m7	F7	G^{b}maj7#5	Adim7	B^{b}maj7
Superphrygian	Cm7	D^{b}m7	E^{b}7	F^{b}maj7#5	Gdim7	A^{b}maj7	B^{b}m7b5
Lydian b3	Cm7	D7	E^{b}maj7#5	F$^\#$dim7	Gmaj7	Am7b5	Bm7
Mixolydian b2	C7	D^{b}maj7#5	Edim7	Fmaj7	Gm7b5	Am7	B^{b}m7
Lydian Augmented #2	Cmaj7#5	D$^\#$dim7	Emaj7	F$^\#$m7b5	G$^\#$m7	Am7	B7
Locrian bb7	Cdim7	D^{b}maj7	E^{b}m7b5	Fm7	G^{b}m7	A^{b}7	B^{bb}maj7#5
Double Harmonic	Cmaj7	D^{b}maj7	Em^{dim7}	Fm$^\Delta$	G7b5	A^{b}maj7#5	B(?)
Lydian #2 #6	Cmaj7	D$^\#$m^{dim7}	Em$^\Delta$	F$^\#$7b5	Gmaj7#5	A$^\#$(?)	Bmaj7
Ultraphrygian	Cm^{dim7}	D^{b}m$^\Delta$	E^{b}7b5	F^{b}maj7#5	G(?)	A^{b}maj7	B^{bb}maj7
Hungarian Minor	Cm$^\Delta$	D7b5	E^{b}maj7#5	F$^\#$(?)	Gmaj7	A^{b}maj7	Bm^{dim7}
Oriental	C7b5	D^{b}maj7#5	E(?)	Fmaj7	G^{b}maj7	Am^{dim7}	B^{b}m$^\Delta$
Ionian Augmented #2	Cmaj7#5	D$^\#$(?)	Emaj7	Fmaj7	G$^\#$m^{dim7}	Am$^\Delta$	B7b5
Locrian bb3 bb7	C(?)	D^{b}maj7	E^{bb}maj7	Fm^{dim7}	G^{b}m$^\Delta$	A^{b}7b5	B^{bb}maj7#5

Similitude Substitution – Any Augmented Major Seventh Chord can be approximately identified with a Dominant Seventh Chord (provided with the flat thirteenth) distant an ascending major third. Any Minor Major Seventh Chord can be approximately identified with a Dominant Seventh Chord (provided with the sharp eleventh) distant an ascending perfect fourth [3] [4].

From now onwards, the examined chord (the one which must undergo substitution) will be denoted by a_n. The analysis must be carried out by starting from the final chord of the progression.

MAJOR Key

$$a_n = Ymaj7\#5 \equiv Y^{\Delta}\#5$$

$$s_1^{Ion}maj7\#5 = h_1^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_1^{Ion}$$

$$\langle s_1^{Ion} + \frac{t}{2}\rangle\, maj7\#5 = h_2^{Dorian\ b2} \xleftarrow{mod.\ int.} h_2^{Ion}$$

$$s_2^{Ion}maj7\#5 \xleftarrow{sim.} \langle s_4^{Ion} + \frac{t}{2}\rangle\, 7^{b13} \cong \langle s_4^{Ion} + \frac{t}{2}\rangle\, 7$$

$$\langle s_2^{Ion} + \frac{t}{2}\rangle\, maj7\#5 = h_3^{Ipo\ Ionian} \xleftarrow{mod.\ int.} h_3^{Ion}$$

$$s_3^{Ion}maj7\#5 = h_4^{Super\ Locrian} \xleftarrow{mod.\ int.} h_4^{Ion}$$

$$s_4^{Ion}maj7\#5 \xleftarrow{sim.} s_6^{Ion}7^{b13} \cong s_6^{Ion}7$$

$$\langle s_4^{Ion} + \frac{t}{2}\rangle\, maj7\#5 = h_5^{Locrian\ \#2} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$s_5^{Ion}maj7\#5 = h_5^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$\langle s_5^{Ion} + \frac{t}{2}\rangle\, maj7\#5 = h_6^{Mixolydian\ b6} \xleftarrow{mod.\ int.} h_6^{Ion}$$

$$s_6^{Ion}maj7\#5 = h_7^{Locrian\ bb7} \xleftarrow{mod.\ int.} h_7^{Ion}$$

$$\langle s_6^{Ion} + \frac{t}{2}\rangle\, maj7\#5 = h_7^{Lydian\ Dominant} \xleftarrow{mod.\ int.} h_7^{Ion}$$

$$s_7^{Ion}maj7\#5 \xleftarrow{sim.} \langle s_2^{Ion} + \frac{t}{2}\rangle\, 7^{b13} \cong \langle s_2^{Ion} + \frac{t}{2}\rangle\, 7$$

MINOR Key

$$a_n = Ymaj7\#5 \equiv Y^{\Delta}\#5$$

$$s_1^{Aeo}maj7\#5 = h_1^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_1^{Aeo}$$

$$\langle s_1^{Aeo} + \frac{t}{2}\rangle\, maj7\#5 = h_2^{Dorian\ b2} \xleftarrow{mod.\ int.} h_2^{Aeo}$$

$$s_2^{Aeo}maj7\#5 \xleftarrow{sim.} \langle s_4^{Aeo} + \frac{t}{2}\rangle\, 7^{b13} \cong \langle s_4^{Aeo} + \frac{t}{2}\rangle\, 7$$

$$s_3^{Aeo}maj7\#5 = h_3^{Ipo\ Ionian} \xleftarrow{mod.\ int.} h_3^{Aeo}$$

$$\langle s_3^{Aeo} + \frac{t}{2}\rangle\, maj7\#5 = h_4^{Super\ Locrian} \xleftarrow{mod.\ int.} h_4^{Aeo}$$

$$s_4^{Aeo}maj7\#5 \xleftarrow{sim.} \langle s_6^{Aeo} + \frac{t}{2}\rangle\, 7^{b13} \cong \langle s_6^{Aeo} + \frac{t}{2}\rangle\, 7$$

$$\langle s_4^{Aeo} + \frac{t}{2}\rangle\, maj7\#5 = h_5^{Locrian\ \#2} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$s_5^{Aeo}maj7\#5 = h_5^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$s_6^{Aeo}maj7\#5 = h_6^{Mixolydian\ b6} \xleftarrow{mod.\ int.} h_6^{Aeo}$$

$$\langle s_6^{Aeo} + \frac{t}{2}\rangle\, maj7\#5 = h_7^{Locrian\ bb7} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

$$s_7^{Ion}maj7\#5 = h_7^{Lydian\ Dominant} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

$$\langle s_7^{Aeo} + \frac{t}{2}\rangle\, maj7\#5 \xleftarrow{sim.} s_3^{Aeo}7^{b13} \cong s_3^{Aeo}7$$

4. Inverse Substitutions (Modal Interchange and Similitude)* involving MINOR MAJOR SEVENTH CHORDS

*See *point 3* for the definitions of *Modal Interchange* and *Similitude Substitution*.

MAJOR Key

$$a_n = Ym^{\Delta}$$

$$s_1^{Ion}m^{\Delta} = h_1^{Ipo\ Ionian} \xleftarrow{mod.\ int.} h_1^{Ion}$$

$$\langle s_1^{Ion} + \frac{t}{2}\rangle\, m^{\Delta} = h_2^{Super\ Locrian} \xleftarrow{mod.\ int.} h_2^{Ion}$$

$$s_2^{Ion}m^{\Delta} \xleftarrow{sim.} h_5^{Ion}\,{}^{\#11}_{9} \cong h_5^{Ion}$$

$$\langle s_2^{Ion} + \frac{t}{2}\rangle\, m^{\Delta} = h_3^{Locrian\ \#2} \xleftarrow{mod.\ int.} h_3^{Ion}$$

$$s_3^{Ion}m^{\Delta} = h_3^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_3^{Ion}$$

$$s_4^{Ion}m^{\Delta} = h_4^{Mixolydian\ b6} \xleftarrow{mod.\ int.} h_4^{Ion}$$

$$\langle s_4^{Ion} + \frac{t}{2}\rangle\, m^{\Delta} = h_5^{Locrian\ bb3\ bb7} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$s_5^{Ion}m^{\Delta} = h_5^{Lydian\ Dominant} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$\langle s_5^{Ion} + \frac{t}{2}\rangle\, m^{\Delta} \xleftarrow{sim.} \langle s_1^{Ion} + \frac{t}{2}\rangle\, 7\,{}^{\#11}_{9} \cong \langle s_1^{Ion} + \frac{t}{2}\rangle\, 7$$

$$s_6^{Ion}m^{\Delta} = h_6^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_6^{Ion}$$

$$\langle s_6^{Ion} + \frac{t}{2}\rangle\, m^{\Delta} = h_7^{Dorian\ b2} \xleftarrow{mod.\ int.} h_7^{Ion}$$

$$s_7^{Ion}m^{\Delta} \xleftarrow{sim.} s_3^{Ion}7\,{}^{\#11}_{9} \cong s_3^{Ion}7$$

MINOR Key

$$a_n = Ym^{\Delta}$$

$$s_1^{Aeo}m^{\Delta} = h_1^{Ipo\ Ionian} \xleftarrow{mod.\ int.} h_1^{Aeo}$$

$$\langle s_1^{Aeo} + \frac{t}{2}\rangle\, m^{\Delta} = h_2^{Super\ Locrian} \xleftarrow{mod.\ int.} h_2^{Aeo}$$

$$s_2^{Aeo}m^{\Delta} \xleftarrow{sim.} s_5^{Aeo}\,{}^{\#11}_{9} \cong s_5^{Aeo}7$$

$$s_3^{Aeo}m^{\Delta} = h_3^{Locrian\ \#2} \xleftarrow{mod.\ int.} h_3^{Aeo}$$

$$\langle s_3^{Aeo} + \frac{t}{2}\rangle\, m^{\Delta} = h_3^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_3^{Aeo}$$

$$s_4^{Aeo}m^{\Delta} = h_4^{Mixolydian\ b6} \xleftarrow{mod.\ int.} h_4^{Aeo}$$

$$\langle s_4^{Aeo} + \frac{t}{2}\rangle\, m^{\Delta} = h_5^{Locrian\ bb3\ bb7} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$s_5^{Aeo}m^{\Delta} = h_5^{Lydian\ Dominant} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$s_6^{Aeo}m^{\Delta} \xleftarrow{sim.} \langle s_1^{Aeo} + \frac{t}{2}\rangle\, 7\,{}^{\#11}_{9} \cong \langle s_1^{Aeo} + \frac{t}{2}\rangle\, 7$$

$$\langle s_6^{Aeo} + \frac{t}{2}\rangle\, m^{\Delta} = h_6^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_6^{Aeo}$$

$$s_7^{Aeo}m^{\Delta} = h_7^{Dorian\ b2} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

$$\langle s_7^{Aeo} + \frac{t}{2}\rangle\, m^{\Delta} \xleftarrow{sim.} \langle s_3^{Aeo} + \frac{t}{2}\rangle\, 7\,{}^{\#11}_{9} \cong \langle s_3^{Aeo} + \frac{t}{2}\rangle\, 7$$

5. Inverse Substitutions (Modal Interchange* and Inverse Diminished Substitution) involving DIMINISHED CHORDS followed by Half-Diminished or Diminished Chords

Diminished Substitution – Any Dominant Seventh Chord, especially if provided with the flat ninth, can be replaced, even if it were to arise from a previous harmonic substitution, by a Diminished Chord distant a major third, a perfect fifth, a minor seventh or a flat ninth from the initial chord. All the above-mentioned intervals must be regarded as ascending [1] [2] [3] [4] [7] [8].

In other terms, if we denote with P a generic note belonging to the *Chromatic* scale, we can write, with obvious meaning of the notation, as follows:

$$P7^{(b9)} \xrightarrow{dim.} Ydim7$$

$$Y = P + 2t + \frac{3n}{2}t \qquad n = 0,1,2,3$$

More explicitly, we have:

$$P7^{(b9)} \xrightarrow{dim.} \begin{cases} \langle P + 2t \rangle dim7 & n = 0, major\ third \\ \langle P + \frac{7}{2}t \rangle\, dim7 & n = 1, perfect\ fifth \\ \langle P + 5t \rangle dim7 & n = 2, minor\ seventh \\ \langle P + \frac{13}{2}t \rangle\, dim7 & n = 3, flat\ ninth \end{cases}$$

*See *point 3* for the definition of *Modal Interchange*

MAJOR Key

$$\begin{cases} a_n = Ydim7 \\ a_{n+1} = Zm7b5, Zdim7 \end{cases}$$

$$s_1^{Ion} dim7 = h_1^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_1^{Ion}$$

$$\langle s_1^{Ion} + \frac{t}{2} \rangle\, dim7 \xleftarrow{dim.} s_6^{Ion} 7^{b9} \cong s_6^{Ion} 7$$

$$s_2^{Ion} dim7 \xleftarrow{dim.} \langle s_6^{Ion} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_6^{Ion} + \frac{t}{2} \rangle\, 7$$

$$\langle s_2^{Ion} + \frac{t}{2} \rangle\, dim7 = h_2^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_2^{Ion}$$

$$s_3^{Ion} dim7 = h_3^{Phrygian\ Dominant} \xleftarrow{mod.\ int.} h_3^{Ion}$$

$$s_4^{Ion} dim7 \xleftarrow{dim.} \langle s_1^{Ion} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_1^{Ion} + \frac{t}{2} \rangle\, 7$$

$$\langle s_4^{Ion} + \frac{t}{2} \rangle\, dim7 = h_4^{Romanian} \xleftarrow{mod.\ int.} h_4^{Ion}$$

$$s_5^{Ion} dim7 = h_5^{Super\ Phrygian} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$\langle s_5^{Ion} + \frac{t}{2} \rangle\, dim7 = h_5^{Ionian\ Augmented} \xleftarrow{mod.\ int.} h_5^{Ion}$$

$$s_6^{Ion} dim7 = h_6^{Locrian\ \#6} \xleftarrow{mod.\ int.} h_6^{Ion}$$

$$\langle s_6^{Ion} + \frac{t}{2} \rangle\, dim7 \xleftarrow{dim.} \langle s_4^{Ion} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_4^{Ion} + \frac{t}{2} \rangle\, 7$$

$$s_7^{Ion} dim7 = h_6^{Harmonic\ Minor} \xleftarrow{mod.\ int.} h_7^{Ion}$$

MINOR Key

$$\begin{cases} a_n = Ydim7 \\ a_{n+1} = Zm7b5, Zdim7 \end{cases}$$

$$s_1^{Aeo} dim7 = h_1^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_1^{Aeo}$$

$$\langle s_1^{Aeo} + \frac{t}{2} \rangle\, dim7 \xleftarrow{dim.} \langle s_6^{Aeo} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_6^{Aeo} + \frac{t}{2} \rangle\, 7$$

$$s_2^{Aeo} dim7 \xleftarrow{dim.} s_7^{Aeo\,b9} \cong s_7^{Aeo} 7$$

$$s_3^{Aeo} dim7 = h_2^{Lydian\ \#2} \xleftarrow{mod.\ int.} h_2^{Aeo}$$

$$\langle s_3^{Aeo} + \frac{t}{2} \rangle\, dim7 = h_3^{Phrygian\ Dominant} \xleftarrow{mod.\ int.} s_3^{Aeo}$$

$$s_4^{Aeo} dim7 \xleftarrow{dim.} \langle s_1^{Aeo} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_1^{Aeo} + \frac{t}{2} \rangle\, 7$$

$$\langle s_4^{Aeo} + \frac{t}{2} \rangle\, dim7 = h_4^{Romanian} \xleftarrow{mod.\ int.} h_4^{Aeo}$$

$$s_5^{Aeo} dim7 = h_5^{Super\ Phrygian} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$s_6^{Aeo} dim7 = h_5^{Ionian\ Augmented} \xleftarrow{mod.\ int.} h_5^{Aeo}$$

$$\langle s_6^{Aeo} + \frac{t}{2} \rangle\, dim7 = h_6^{Locrian\ \#6} \xleftarrow{mod.\ int.} h_6^{Aeo}$$

$$s_7^{Aeo} dim7 \xleftarrow{dim.} \langle s_4^{Aeo} + \frac{t}{2} \rangle\, 7^{b9} \cong \langle s_4^{Aeo} + \frac{t}{2} \rangle\, 7$$

$$\langle s_7^{Aeo} + \frac{t}{2} \rangle\, dim7 = h_6^{Harmonic\ Minor} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

6. Inverse Diminished Substitutions involving DIMINISHED CHORDS not followed by Half-Diminished or Diminished Chords

*See *point 5* for the definition of *Diminished Substitution*

Unlike the previous version of *CAT* [3] [4], we herein consider two cases altogether:

6.1. Inverse Diminished Substitutions involving Diminished chords followed by Dominant Seventh Chords

$$a_n = Ydim7$$

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}[Z7] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} Z7$$

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{5}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} \langle Z + \frac{5}{2}t\rangle 7$$

$$\begin{cases} a_{n+1} = Z7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{7}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} \langle Z + \frac{7}{2}t\rangle 7$$

6.2. Inverse Diminished Substitutions involving Diminished chords followed by Major Seventh or Minor Seventh Chords

$$a_n = Ydim7$$

$$\begin{cases} a_{n+1} = Zmaj7, Zm7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{5}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} \langle Z + \frac{5}{2}t\rangle 7$$

$$\begin{cases} a_{n+1} = Zmaj7, Zm7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{7}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} \langle Z + \frac{7}{2}t\rangle 7$$

$$\begin{cases} a_{n+1} = Zmaj7, Zm7 \\ a_n \in sub^{dim}\left[\langle Z + \frac{9}{2}t\rangle 7\right] \end{cases} \Longrightarrow a_n \overset{dim.}{\longleftarrow} \langle Z + \frac{9}{2}t\rangle 7$$

7. CONTRACTION (Inverse EXPANSION)

Expansion – Any Dominant Seventh Chord, by forgoing half of its duration, can be imagined as being preceded by a Minor Seventh or a Half-Diminished Chord distant a descending perfect fourth and characterized by a duration equal to half of the one of the initial chord. In other terms, any Dominant Seventh Chord can be converted into a half-cadence [3] [4].

$$\begin{cases} a_n = Ym7, Ym7b5 \\ a_{n+1} = \langle Y + \frac{5}{2}t\rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n | a_{n+1} \overset{exp.}{\longleftarrow} a_{n+1} | a_{n+1}$$

8. STRUCTURE REDUCTION*

Note – Every time a Reduction has been carried out, it is necessary to evaluate the possibility of carry out Contractions the application of which, in turn, could make feasible another Structure Reduction, and so on. In other terms, we have a LOOP that can be stopped only if there are no conditions that make possible further Contractions and Structure Reductions [3] [4].

*See *point 2* for the definition of *Structure Reduction*

9. Inverse Substitutions (Modal Interchange*) involving MAJOR SEVENTH CHORDS different from h_1 and h_4, if the Key is Major, or h_3 and h_6, if the Key is Minor (Major Seventh Chords that does not belong to the Harmonization Vector).

*See *point 3* for the definition of *Modal Interchange*

MAJOR Key

$$a_n = Ymaj7 \equiv Y^{\Delta} \neq h_1^{Ion}, h_4^{Ion}$$

$$\langle s_1^{Ion} + \frac{t}{2}\rangle maj7 = h_2^{Phrygian} \overset{mod.\ int.}{\longleftarrow} h_2^{Ion}$$

$$s_2^{Ion} maj7 = h_3^{Locrian\ bb3\ bb7} \overset{mod.\ int.}{\longleftarrow} h_3^{Ion}$$

MINOR Key

$$a_n = Ymaj7 \equiv Y^{\Delta} \neq h_3^{Aeo}, h_6^{Aeo}$$

$$s_1^{Aeo} maj7 = h_1^{Ionian} \overset{mod.\ int.}{\longleftarrow} h_1^{Aeo}$$

$$\langle s_1^{Aeo} + \frac{t}{2}\rangle maj7 = h_2^{Phrygian} \overset{mod.\ int.}{\longleftarrow} h_2^{Aeo}$$

$$\langle s_2^{Ion} + \frac{t}{2}\rangle maj7 = h_3^{Dorian} \xleftarrow{mod.\ int.} h_3^{Ion}$$
$$s_3^{Ion} maj7 = h_3^{Lydian\ Augmented\ \#2} \xleftarrow{mod.\ int.} h_3^{Ion}$$
$$\langle s_4^{Ion} + \frac{t}{2}\rangle maj7 = h_5^{Locrian} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$s_5^{Ion} maj7 = h_5^{Lydian} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$\langle s_5^{Ion} + \frac{t}{2}\rangle maj7 = h_6^{Phrygian} \xleftarrow{mod.\ int.} h_6^{Ion}$$
$$s_6^{Ion} maj7 = h_7^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_7^{Ion}$$
$$\langle s_6^{Ion} + \frac{t}{2}\rangle maj7 = h_7^{Dorian} \xleftarrow{mod.\ int.} h_7^{Ion}$$
$$s_7^{Ion} maj7 = h_7^{Lydian\ \#2\ \#6} \xleftarrow{mod.\ int.} h_7^{Ion}$$

$$s_2^{Aeo} maj7 = h_3^{Locrian\ bb3\ bb7} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$\langle s_3^{Aeo} + \frac{t}{2}\rangle maj7 = h_3^{Lydian\ Augmented\ \#2} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$s_4^{Aeo} maj7 = h_4^{Ionian} \xleftarrow{mod.\ int.} h_4^{Aeo}$$
$$\langle s_4^{Aeo} + \frac{t}{2}\rangle maj7 = h_5^{Locrian} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$s_5^{Aeo} maj7 = h_5^{Lydian} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$\langle s_6^{Aeo} + \frac{t}{2}\rangle maj7 = h_7^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$
$$s_7^{Aeo} maj7 = h_7^{Dorian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$
$$\langle s_7^{Aeo} + \frac{t}{2}\rangle maj7 = h_7^{Lydian\ \#2\ \#6} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

10. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

11. Inverse Substitutions (Modal Interchange and Similitude)* involving HALF – DIMINISHED CHORDS different from h_7, if the Key is Major, or h_2, if the Key is Minor (Half – Diminished Chords that does not belong to the Harmonization Vector).

*See *point 3* for the definitions of *Modal Interchange* and *Similitude*

MAJOR Key

$$a_n = Ym7b5 \equiv Y\emptyset \neq h_7^{Ion}$$

$$\begin{cases} a_n = \langle s_4^{Ion} + \frac{1}{2}t\rangle m7b5 \\ a_{n+1} = s_4^{Ion}7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n\ (no\ substitution)$$

$$\begin{cases} a_n = s_i^{Ion} m7b5 \\ a_{n+1} = \langle s_i^{Ion} + \frac{11}{2}t\rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n\ (no\ substitution)\quad i \neq 4{,}7$$

$$s_1^{Ion} m7b5 = h_1^{Locrian} \xleftarrow{mod.\ int.} h_1^{Ion}$$
$$\langle s_1^{Ion} + \frac{t}{2}\rangle m7b5 \xleftarrow{sim} s_6^{Ion}7^9 \cong s_6^{Ion}7$$
$$s_2^{Ion} m7b5 = h_2^{Aeolian} \xleftarrow{mod.\ int.} h_2^{Ion}$$
$$\langle s_2^{Ion} + \frac{t}{2}\rangle m7b5 = h_3^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_3^{Ion}$$
$$s_3^{Ion} m7b5 = h_3^{Mixolydian} \xleftarrow{mod.\ int.} h_3^{Ion}$$
$$s_4^{Ion} m7b5 \xleftarrow{sim.} \langle s_1^{Ion} + \frac{t}{2}\rangle 7^9 \cong \langle s_1^{Ion} + \frac{t}{2}\rangle 7$$
$$\langle s_4^{Ion} + \frac{t}{2}\rangle m7b5 = h_4^{Lydian} \xleftarrow{mod.\ int.} h_4^{Ion}$$
$$s_5^{Ion} m7b5 = h_5^{Phrygian} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$\langle s_5^{Ion} + \frac{t}{2}\rangle m7b5 = h_5^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$s_6^{Ion} m7b5 = h_6^{Dorian} \xleftarrow{mod.\ int.} h_6^{Ion}$$
$$\langle s_6^{Ion} + \frac{t}{2}\rangle m7b5 = h_7^{Super\ Locrian} \xleftarrow{mod.\ int.} h_7^{Ion}$$

MINOR Key

$$a_n = Ym7b5 \equiv Y\emptyset \neq h_2^{Aeo}$$

$$\begin{cases} a_n = \langle s_6^{Aeo} + \frac{1}{2}t\rangle m7b5 \\ a_{n+1} = s_6^{Aeo}7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n\ (no\ substitution)$$

$$\begin{cases} a_n = s_i^{Aeo} m7b5 \\ a_{n+1} = \langle s_i^{Aeo} + \frac{11}{2}t\rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n\ (no\ substitution)\quad i \neq 2{,}6$$

$$s_1^{Aeo} m7b5 = h_1^{Locrian} \xleftarrow{mod.\ int.} h_1^{Aeo}$$
$$\langle s_1^{Aeo} + \frac{t}{2}\rangle m7b5 \xleftarrow{sim} \langle s_6^{Aeo} + \frac{t}{2}\rangle 7^9 \cong \langle s_6^{Aeo} + \frac{t}{2}\rangle 7$$
$$s_3^{Aeo} m7b5 = h_3^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$\langle s_3^{Aeo} + \frac{t}{2}\rangle m7b5 = h_3^{Mixolydian} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$s_4^{Aeo} m7b5 \xleftarrow{sim.} \langle s_1^{Aeo} + \frac{t}{2}\rangle 7^9 \cong \langle s_1^{Aeo} + \frac{t}{2}\rangle 7$$
$$\langle s_4^{Aeo} + \frac{t}{2}\rangle m7b5 = h_4^{Lydian} \xleftarrow{mod.\ int.} h_4^{Aeo}$$
$$s_5^{Aeo} m7b5 = h_5^{Phrygian} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$s_6^{Aeo} m7b5 = h_5^{Lydian\ Augmented} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$\langle s_6^{Aeo} + \frac{t}{2}\rangle m7b5 = h_6^{Dorian} \xleftarrow{mod.\ int.} h_6^{Aeo}$$
$$s_7^{Aeo} m7b5 = h_7^{Super\ Locrian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$
$$\langle s_7^{Aeo} + \frac{t}{2}\rangle m7b5 = h_7^{Ionian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

12. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

13. Inverse Substitutions (Modal Interchange*) involving MINOR SEVENTH CHORDS different from h_2, h_3, and h_6, if the Key is Major, or h_1, h_4, and h_5, if the Key is Minor (Minor Seventh Chords that does not belong to the Harmonization Vector).

*See *point 3* for the definition of *Modal Interchange*

MAJOR Key

$$a_n = Ym7 \neq h_2^{Ion}, h_3^{Ion}, h_6^{Ion}$$

$$\begin{cases} a_n = \langle s_4^{Ion} + \frac{1}{2}t \rangle m7 \\ a_{n+1} = s_4^{Ion}7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n \ (no\ substitution)$$

$$\begin{cases} a_n = s_i^{Ion}m7 \\ a_{n+1} = \langle s_i^{Ion} + \frac{11}{2}t \rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n \ (no\ substitution) \quad i = 1,5,7$$

$$s_1^{Ion}m7 = h_1^{Aeolian} \xleftarrow{mod.\ int.} h_1^{Ion}$$
$$\langle s_1^{Ion} + \frac{t}{2} \rangle m7 = h_2^{Super\ Phrygian} \xleftarrow{mod.\ int.} h_2^{Ion}$$
$$\langle s_2^{Ion} + \frac{t}{2} \rangle m7 = h_3^{Locrian} \xleftarrow{mod.\ int.} h_3^{Ion}$$
$$s_4^{Ion}m7 = h_4^{Aeolian} \xleftarrow{mod.\ int.} h_4^{Ion}$$
$$\langle s_4^{Ion} + \frac{t}{2} \rangle m7 = h_5^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$s_5^{Ion}m7 = h_5^{Aeolian} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$\langle s_5^{Ion} + \frac{t}{2} \rangle m7 = h_5^{Lydian\ Augmented\ \#2} \xleftarrow{mod.\ int.} h_5^{Ion}$$
$$\langle s_6^{Ion} + \frac{t}{2} \rangle m7 = h_7^{Phrygian} \xleftarrow{mod.\ int.} h_7^{Ion}$$
$$s_7^{Ion}m7 = h_7^{Lydian} \xleftarrow{mod.\ int.} h_7^{Ion}$$

MINOR Key

$$a_n = Ym7 \neq h_1^{Aeo}, h_4^{Aeo}, h_5^{Aeo}$$

$$\begin{cases} a_n = \langle s_6^{Aeo} + \frac{1}{2}t \rangle m7 \\ a_{n+1} = s_6^{Aeo}7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n \ (no\ substitution)$$

$$\begin{cases} a_n = s_i^{Aeo}m7 \\ a_{n+1} = \langle s_i^{Aeo} + \frac{11}{2}t \rangle 7 \\ a_n, a_{n+1} \in bar_k \\ T(a_n) = T(a_{n+1}) \\ beat(a_n) = on \end{cases} \Longrightarrow a_n = a_n \ (no\ substitution) \quad i = 2,3,7$$

$$\langle s_1^{Aeo} + \frac{t}{2} \rangle m7 = h_2^{Super\ Phrygian} \xleftarrow{mod.\ int.} h_2^{Aeo}$$
$$s_2^{Aeo}m7 = h_2^{Ionian} \xleftarrow{mod.\ int.} h_2^{Aeo}$$
$$s_3^{Aeo}m7 = h_3^{Locrian} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$\langle s_3^{Aeo} + \frac{t}{2} \rangle m7 = h_3^{Ionian} \xleftarrow{mod.\ int.} h_3^{Aeo}$$
$$\langle s_4^{Aeo} + \frac{t}{2} \rangle m7 = h_5^{Ultra\ Locrian} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$s_6^{Aeo}m7 = h_5^{Lydian\ Augmented\ \#2} \xleftarrow{mod.\ int.} h_5^{Aeo}$$
$$\langle s_6^{Aeo} + \frac{t}{2} \rangle m7 = h_2^{Ionian} \xleftarrow{mod.\ int.} h_6^{Aeo}$$
$$s_7^{Aeo}m7 = h_7^{Phrygian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$
$$\langle s_7^{Aeo} + \frac{t}{2} \rangle m7 = h_7^{Lydian} \xleftarrow{mod.\ int.} h_7^{Aeo}$$

14. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

15. (Inverse) TRITONE Substitutions

Tritone Substitution – Any Dominant Seventh Chord, especially if altered, can be replaced, even if it were to arise from a previous harmonic substitution, by a chord of the same kind (a Dominant Seventh Chord) distant three whole tones from the initial chord.

Generally, if we denote with P a generic note belonging to the *Chromatic* scale, and with t a whole tone interval, we can write:

$$X7 \xrightarrow{dim.} Y7$$

$$Y = X + 3t$$

MAJOR Key

$$a_n = Y7$$

$$\begin{cases} a_n = Y7 \\ Y \neq s_i^{Ion} \end{cases} \Longrightarrow a_n \xleftarrow{tri.} \langle Y + 3t \rangle 7 \qquad i = 1, \dots, 7$$

MINOR Key

$$a_n = Y7$$

$$\begin{cases} a_n = Y7 \\ Y \neq s_i^{Aeo} \end{cases} \Longrightarrow a_n \xleftarrow{tri.} \langle Y + 3t \rangle 7 \qquad i = 1, \dots, 7$$

$$\begin{cases} a_n = s_4^{Ion}7 \\ a_{n-1} = h_5^{Ion}, \langle s_5^{Ion} + 3t \rangle 7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_7^{Ion}7$$

$$\begin{cases} a_n = s_4^{Ion}7 \\ a_{n-1} = \langle s_4^{Ion} + \frac{1}{2}t \rangle\, m7b5, \langle s_4^{Ion} + \frac{1}{2}t \rangle\, m7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_7^{Ion}7$$

$$\begin{cases} a_n = s_7^{Ion}7 \\ a_{n-1} = s_1^{Ion}m7b5, s_1^{Ion}m7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_4^{Ion}7$$

$$\begin{cases} a_n = s_2^{Aeo}7 \\ a_{n-1} = h_5^{Aeo}, s_5^{Aeo}7, \langle s_5^{Aeo} + 3t \rangle 7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_6^{Aeo}7$$

$$\begin{cases} a_n = s_2^{Aeo}7 \\ a_{n-1} = s_3^{Aeo}m7b5, s_3^{Aeo}m7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_6^{Aeo}7$$

$$\begin{cases} a_n = s_6^{Aeo}7 \\ a_{n-1} = \langle s_6^{Aeo} + \frac{1}{2}t \rangle\, m7b5, \langle s_6^{Ion} + \frac{1}{2}t \rangle\, m7 \end{cases} \Longrightarrow a_n \xleftarrow{tri.} s_2^{Aeo}7$$

16. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

17. SECONDARY DOMINANTS Inverse Substitutions

Secondary Dominant Substitution – Any chord, even if it were to arise from a previous harmonic substitution, can be converted into a Dominant Seventh Chord [1] [2] [3] [4] [7] [8].

MAJOR Key

$a_n = Y7$

$$a_n = s_i^{Ion}7 \Longrightarrow a_n \xleftarrow{sec.\ dom.} h_i^{Ion} \qquad i \neq 5$$

MINOR Key

$a_n = Y7$

$$a_n = s_i^{Aeo}7 \Longrightarrow a_n \xleftarrow{sec.\ dom.} h_i^{Aeo} \qquad i \neq 5,7$$

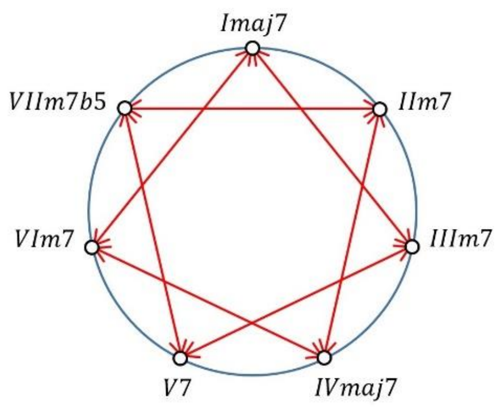
The Secondary Dominants Inverse Substitutions involving s_5 are procrastinated in order to facilitate possible Contractions.

18. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

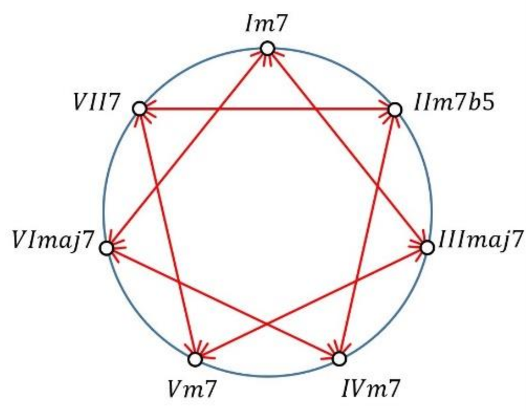
19. (Inverse) DIATONIC Substitutions

Diatonic Substitution – Two chords that arise from the harmonization of the same scale are interchangeable if the distance between them (between the roots) is equal to a diatonic third (both ascending and descending) [1] [2] [3] [4] [7] [8].

Substitutions Chart – MAJOR Key



Substitutions Chart – MINOR Key



19.1. (Inverse) DIATONIC Substitutions involving h_6

MAJOR Key

$$a_n = h_6^{Ion} \Longrightarrow a_n \xleftarrow{dia.} h_1^{Ion}$$

MINOR Key

$$a_n = h_6^{Aeo} \Longrightarrow a_n \xleftarrow{dia.} h_1^{Aeo}$$

19.2. (Inverse) DIATONIC Substitutions involving h_3

MAJOR Key

$$\begin{cases} a_n = h_3^{Ion} \\ a_{n+1} = a_{n-1} = h_1^{Ion} \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_1^{Ion}$$

$$\begin{cases} a_n = h_3^{Ion} \\ a_{n+1} \neq h_2^{Ion}, h_4^{Ion} \\ beat(a_n) = off \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Ion}$$

$$\begin{cases} a_n = h_3^{Ion} \\ a_{n+1} = h_2^{Ion}, h_4^{Ion} \\ a_{n+2} = h_5^{Ion} \\ a_{n+1}, a_{n+2} \in bar_k \\ beat(a_n) = off \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Ion}$$

$$\begin{cases} a_n = h_3^{Ion} \\ a_{n+1} \neq h_2^{Ion}, h_4^{Ion} \\ a_{n-1} = h_2^{Ion}, h_4^{Ion}, h_7^{Ion} \\ a_{n-1}, a_n \in bar_k \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Ion}$$

$$otherwise: a_n = h_3^{Ion} \overset{dia.}{\longleftarrow} h_1^{Ion}$$

MINOR Key

$$\begin{cases} a_n = h_3^{Aeo} \\ a_{n+1} = a_{n-1} = h_1^{Aeo} \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_1^{Aeo}$$

$$\begin{cases} a_n = h_3^{Aeo} \\ a_{n+1} \neq h_2^{Aeo}, h_4^{Aeo} \\ beat(a_n) = off \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Aeo}$$

$$\begin{cases} a_n = h_3^{Aeo} \\ a_{n+1} = h_2^{Aeo}, h_4^{Aeo} \\ a_{n+2} = s_5^{Aeo}7 \\ a_{n+1}, a_{n+2} \in bar_k \\ beat(a_n) = off \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Aeo}$$

$$\begin{cases} a_n = h_3^{Aeo} \\ a_{n+1} \neq h_2^{Aeo}, h_4^{Aeo} \\ a_{n-1} = h_2^{Aeo}, h_4^{Aeo}, h_7^{Aeo} \\ a_{n-1}, a_n \in bar_k \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_5^{Aeo}$$

$$otherwise: a_n = h_3^{Aeo} \overset{dia.}{\longleftarrow} h_1^{Aeo}$$

19.3. (Inverse) DIATONIC Substitutions involving h_7

MAJOR Key

$$\begin{cases} a_n = h_7^{Ion} \\ a_{n+1} = h_2^{Ion}, h_4^{Ion} \\ a_{n+2} = h_5^{Ion} \\ beat(a_n) = on \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Ion}$$

$$\begin{cases} a_n = h_7^{Ion} \\ a_{n+1} = h_2^{Ion}, h_4^{Ion} \\ a_{n+2} = h_2^{Ion}, h_4^{Ion} \\ a_{n+3} = h_5^{Ion} \\ beat(a_n) = on \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Ion}$$

$$otherwise: a_n = h_7^{Ion} \overset{dia.}{\longleftarrow} h_5^{Ion}$$

MINOR Key

$$\begin{cases} a_n = h_7^{Aeo} \\ a_{n+1} = h_2^{Aeo}, h_4^{Aeo} \\ a_{n+2} = h_5^{Aeo}, s_5^{Aeo}7 \\ beat(a_n) = on \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Aeo}$$

$$\begin{cases} a_n = h_7^{Aeo} \\ a_{n+1} = h_2^{Aeo}, h_4^{Aeo} \\ a_{n+2} = h_2^{Aeo}, h_4^{Aeo} \\ a_{n+3} = h_5^{Aeo}, s_5^{Aeo}7 \\ beat(a_n) = on \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Aeo}$$

$$otherwise: a_n = h_7^{Ion} \overset{dia.}{\longleftarrow} h_5^{Aeo}$$

19.4. (Inverse) DIATONIC Substitutions involving h_2 and h_4

MAJOR Key

$$\begin{cases} a_n = h_2^{Ion} \\ a_{n+1} = h_1^{Ion}, h_4^{Ion} \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_4^{Ion}$$

$$\begin{cases} a_n = h_4^{Ion} \\ a_{n+1} = h_2^{Ion}, h_5^{Ion} \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Ion}$$

MINOR Key

$$\begin{cases} a_n = h_2^{Aeo} \\ a_{n+1} = h_1^{Aeo}, h_4^{Aeo} \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_4^{Aeo}$$

$$\begin{cases} a_n = h_4^{Aeo} \\ a_{n+1} = h_2^{Aeo}, h_5^{Aeo}, s_5^{Aeo}7 \end{cases} \Longrightarrow a_n \overset{dia.}{\longleftarrow} h_2^{Aeo}$$

20. CONTRACTIONS (Inverse EXPANSION) + STRUCTURE REDUCTION [LOOP]

Note – In this LOOP it is necessary to explicit the chord placed on the third beat.

In detail, a chord covering the beats from the first to the third must be rewritten and treated as two identical chords: the first chord covers the first two beats, the second exclusively covers the third. A chord covering the second and third beats must be rewritten and treated as two identical chords: the first chord covers the second beat, the second covers the third. A chord covering the beats from the second to the fourth must be rewritten and treated as two identical chords: the first chord exclusively covers the second beat, the second covers the last two.

21. Further Inverse SECONDARY DOMINANTS Substitutions

MAJOR Key

All the feasible Secondary Dominants Inverse Substitutions have already been carried out during phase 17.

MINOR Key

$a_n = Y7$

$$a_n = s_5^{Aeo}7 \Longrightarrow a_n \xleftarrow{sec.\ dom.} h_5^{Aeo}$$

EXTRA. Inverse TONICIZATION (to be applied every time it is possible)

Tonicization – Any (major seventh or minor seventh) chord, especially if it covers a certain number of consecutive bars, can be tonicized. The Tonicization, or Micro-Modulation, can be obtained by replacing a portion of the initial chord with a dominant seventh chord distant an ascending perfect fifth, so as to locally create a harmonically Perfect Cadence [1] [2] [3] [4] [7] [8].

MAJOR Key

$$\begin{cases} a_n = Y7 \\ a_{n+1} = a_{n-1} = \langle Y + \frac{5}{2}t \rangle maj7 \\ a_{n+1} = a_{n-1} \neq h_1^{Ion} \\ T(a_n) \leq Min\{T(a_{n+1}), T(a_{n-1})\} \end{cases} \Longrightarrow a_n \xleftarrow{ton.} a_{n+1} = a_{n-1}$$

$$\begin{cases} a_n = Y7 \\ a_{n+1} = a_{n-1} = \langle Y + \frac{5}{2}t \rangle m7 \\ T(a_n) \leq Min\{T(a_{n+1}), T(a_{n-1})\} \end{cases} \Longrightarrow a_n \xleftarrow{ton.} a_{n+1} = a_{n-1}$$

MINOR Key

$$\begin{cases} a_n = Y7 \\ a_{n+1} = a_{n-1} = \langle Y + \frac{5}{2}t \rangle maj7 \\ T(a_n) \leq Min\{T(a_{n+1}), T(a_{n-1})\} \end{cases} \Longrightarrow a_n \xleftarrow{ton.} a_{n+1} = a_{n-1}$$

$$\begin{cases} a_n = Y7 \\ a_{n+1} = a_{n-1} = \langle Y + \frac{5}{2}t \rangle m7 \\ a_{n+1} = a_{n-1} \neq h_1^{Aeo} \\ T(a_n) \leq Min\{T(a_{n+1}), T(a_{n-1})\} \end{cases} \Longrightarrow a_n \xleftarrow{ton.} a_{n+1} = a_{n-1}$$

Remarks

It's worth underlining how chord progression analysis has almost nothing to do with the improvisation built on the original harmonic structure of a song. More precisely, net of a certain "Horizontalization" the musician can exploit by facing specific harmonic aggregates, such as "Tonicizations" and "Turnarounds", the improvisation should be carried out "vertically", abiding to the local harmony of the song.

The Local Tonal Centre can be characterized by significant fluctuations: sometimes, it is very difficult to identify it, in particular when a progression cannot be regarded as being manifestly "tonal" (built on cadences).

And especially in this case, paradoxically, the musician should improvise by abiding to the harmonic progression, chord by chord [13 – 30].

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