



## Computer-aided Harmonic Progression Analysis: CAT CaSe

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Submitted: 6. October 2018  
Published: 8. October 2018  
Volume: 5  
Issue: 8  
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Languages: English  
Keywords: CAT CaSe, Android, Cataldo Advanced Transformations, Chord Progression Analysis, Harmonic Substitutions  
DOI: 10.17160/josha.5.8.477

### Abstract:

In this article we briefly introduce CAT CaSe. CAT CaSe is an innovative Android app, entirely based on an improved version of CAT (Cataldo Advanced Transformations), expressly meant for musicians (professionals and students). The app allows to instantly carry out a detailed analysis of any harmonic progression, without any exception whatsoever. All the phases that constitute the analysis are shown to the user, and the Harmonic Substitutions that occur are accurately discussed.

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# Computer-aided Harmonic Progression Analysis: CAT CaSe

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## Abstract

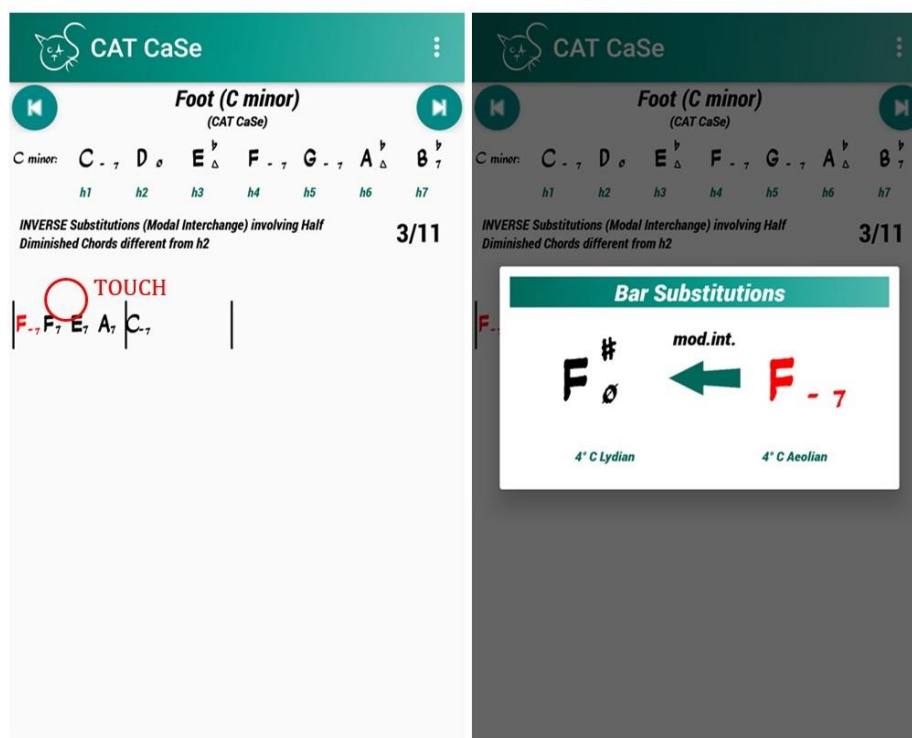
In this article we briefly introduce *CAT CaSe*. *CAT CaSe* is an innovative Android app, entirely based on an improved version of *CAT* (Cataldo Advanced Transformations), expressly meant for musicians (professionals and students). The app allows to instantly carry out a detailed analysis of any harmonic progression, without any exception whatsoever. All the phases that constitute the analysis are shown to the user, and the Harmonic Substitutions that occur are accurately discussed.

## Keywords

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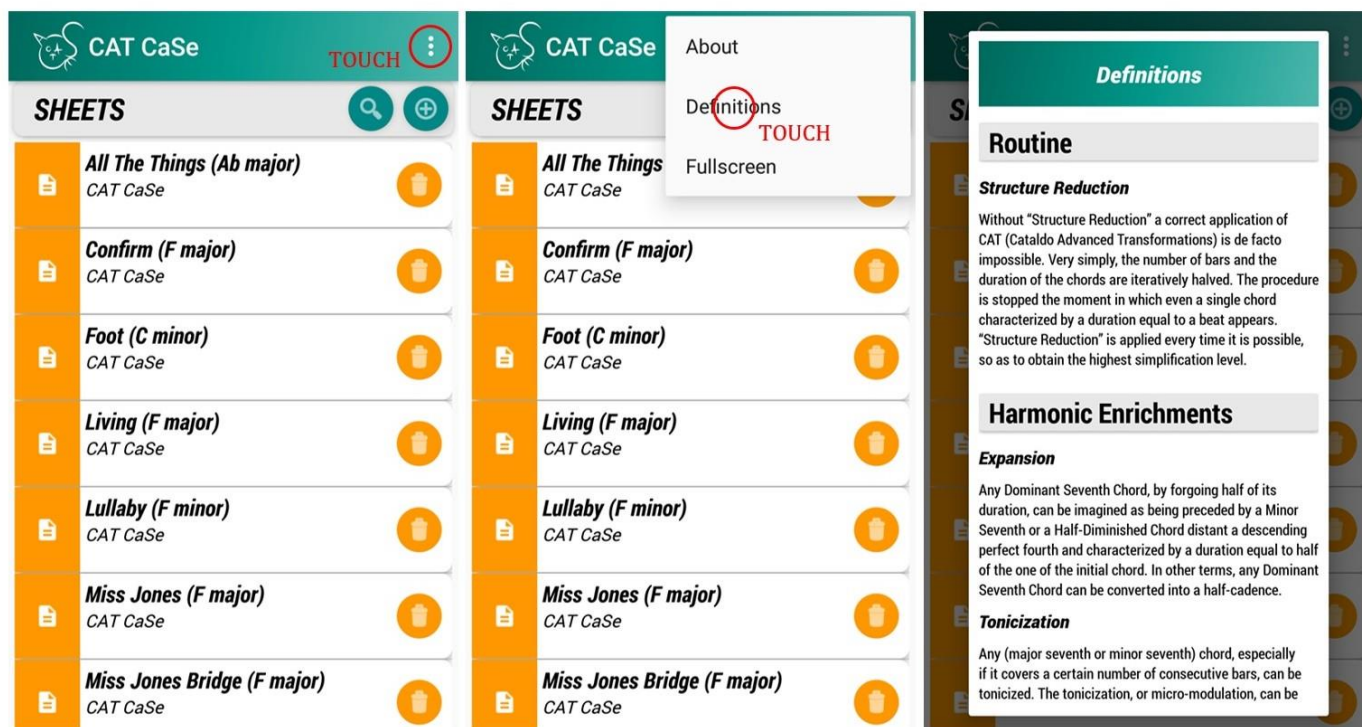
## CAT CaSe

*CAT CaSe* [<https://play.google.com/store/apps/details?id=serra.cataldo.catcase>] is an innovative Android application (developed by Carmine Cataldo e Luigi Serra), entirely based on an improved version of *CAT* (Cataldo Advanced Transformations) [1], expressly meant for musicians (professionals and students). The app allows to instantly carry out a detailed analysis of any harmonic progression, without any exception whatsoever. All the phases that constitute the analysis are shown to the user, and the Harmonic Substitutions [2] [3] [4] [5] that occur are accurately discussed. For instance, every time the app resorts to the so-called "Modal Interchange", the scales between which the parallelism is set are revealed (*CAT CaSe* can compare up to 35 scales/modes), highlighting the degrees which are involved.

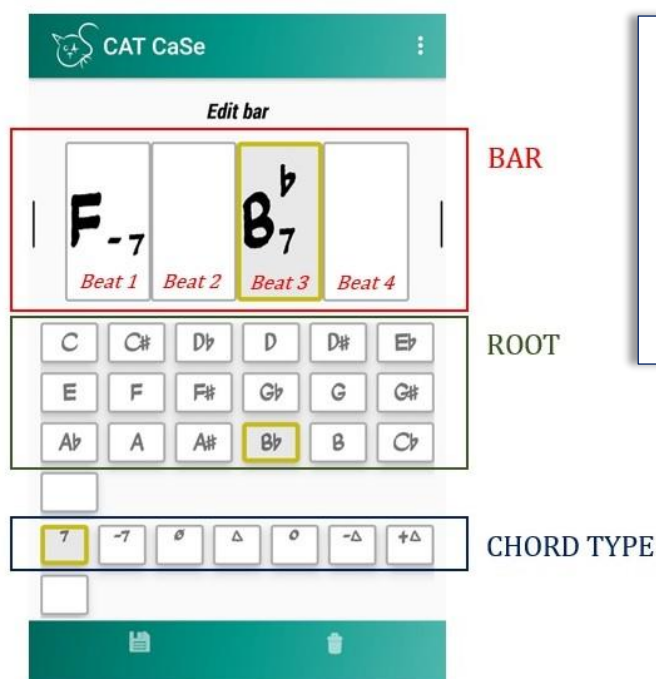


*CAT CaSe*, moreover, can recognize and contextualize possible "Harmonic Enrichments", such as "Tonicizations" and "Expansions" (half-cadences arisen from a single Dominant Seventh Chord). [2] [3] [4] [5]


*CAT CaSe* is provided with a didactic section, finalized to rigorously defining all the Harmonic Substitutions and the Enrichments.



The writing of harmonic progressions is extremely simple and incredibly fast, thanks to the introduction of an innovative interface that allows the user to completely avoid the usage of the alphanumeric keypad. For this reason, *CAT CaSe* turns out to be very useful during live performances, especially if it is necessary to quickly and comprehensibly write, by exploiting an Android device (smartphone or tablet), a harmonic progression the musician must abide to during the accompaniment or the improvisation.



The time signature must always be imagined as being equal to 4/4. For example, even if we deal with a 3/4, we have to consider four pulses per measure (four beats per bar): each beat, in this case, will be characterized by a duration equivalent to a dotted quaver.



After the analysis, *CAT CaSe* allows the user to examine an "inverse procedure", particularly dedicated to musicians interested in composing and arranging. In a few words, starting from the final outcome of the analysis (very often, a sequence of plagal and/or perfect cadences) [4] [5], the app returns, step by step, the initial progression. As a useful additional feature, on the first page of the inverse procedure the parametric version (regardless of the particular key) of the final outcome is also shown.



### CAT (Cataldo Advanced Transformations): Short Overview

CAT CaSe is entirely based on an improved version of CAT (Cataldo Advanced Transformations). A considerable improvement of CAT has been achieved by conducting an extremely thorough analysis of a huge amount of LEGO Bricks (public domain harmonic patterns) [6] [7].

Unlike the previous one [8] [9], the ultimate version of CAT [1] has no limitation concerning the key (any song written in both major and minor key can be analysed), exploits a more rigorous definition of “Similitude”, and takes into consideration “Modal Interchange” and “Tonicization”.

### Examples

#### Example #1 (Major Key): “Inner Urge” (Joe Henderson) – last 8bars

The screenshots show the CAT CaSe interface for the piece "Urge (G major)".

- Initial Progression (1/13):** Shows the Harmonization Vector: G $\Delta$  (h1), A $-7$  (h2), B $-7$  (h3), C $\Delta$  (h4), D $7$  (h5), E $-7$  (h6), F $\sharp$  (h7). The progression is: | E $\Delta$  | D $\flat$  | D $\Delta$  | B $\Delta$  | | C $\Delta$  | A $\Delta$  | B $\flat$  | G $\Delta$  | |
- Structure Reduction (2/13):** Shows the progression: | E $\Delta$  D $\flat$  | D $\Delta$  B $\Delta$  | C $\Delta$  A $\Delta$  | B $\flat$  G $\Delta$  | |
- Structure Reduction (3/13):** Shows the progression: | E $\Delta$  D $\flat$  D $\Delta$  B $\Delta$  | C $\Delta$  A $\Delta$  B $\flat$  G $\Delta$  | |

#### 1. INITIAL STRUCTURE and HARMONIZATION VECTOR

*Harmonization Vector* – The vector whose components represent the seventh chords that arise from the harmonization of the scale (in this case, the *Ionian* of F) [10] [11].

#### 2. STRUCTURE REDUCTION

*Structure Reduction* – Without “Structure Reduction” a correct application of CAT (*Cataldo Advanced Transformations*) is de facto impossible. Very simply, the number of bars and the duration of the chords are iteratively halved. The procedure is stopped the moment in which even a single chord characterized by a duration equal to a beat appears. “Structure Reduction” is applied every time it is possible, so as to obtain the highest simplification level [1] [8] [9].

#### 3. STRUCTURE REDUCTION

\*See point 2 for the definition of *Structure Reduction*



CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: **G**<sub>Δ</sub> **A**<sub>-7</sub> **B**<sub>-7</sub> **C**<sub>Δ</sub> **D**<sub>7</sub> **E**<sub>-7</sub> **F**<sub>♯</sub><sub>♭</sub>

h1 h2 h3 h4 h5 h6 h7

INVERSE Substitutions (Modal Interchange) involving Major Seventh Chords different from h1 and h4 **4/13**

**F**<sub>♯</sub><sub>♭</sub> **D**<sub>7</sub> | **B**<sub>-7</sub> | **C**<sub>Δ</sub> **B**<sub>-7</sub> **B**<sub>♭</sub><sub>7</sub> **G**<sub>Δ</sub> |

4. Inverse Substitutions (Modal Interchange\*) involving MAJOR SEVENTH CHORDS different from  $h_1$  and  $h_4$

*Modal Interchange* – Two chords that arise from the harmonization of two different scales characterized by the same tonic (generic parallel keys) are interchangeable if they are placed in the same position (if they represent the same harmonic degree) [1] [4] [5].

**Bar Substitutions**

**E**<sub>Δ</sub> ← **F**<sub>♯</sub><sub>♭</sub>

( $E \equiv F^b$ )

7° G Ultra Locrian      7° G Ionian

**D**<sub>♭</sub><sub>Δ</sub> ← **D**<sub>7</sub>

5° G Locrian      5° G Ionian

**D**<sub>Δ</sub> ← **D**<sub>7</sub>

5° G Lydian      5° G Ionian

**B**<sub>Δ</sub> ← **B**<sub>-7</sub>

3° G Lydian Augmented #2      3° G Ionian

**Bar Substitutions**

**A**<sub>Δ</sub> ← **B**<sub>-7</sub>

( $A \equiv B^{bb}$ )

3° G Locrian bb3 bb7      3° G Ionian

*Tritone Substitution* – Any Dominant Seventh Chord, especially if altered, can be replaced, even if it were to arise from a previous harmonic substitution, by a chord of the same kind (a Dominant Seventh Chord) distant three whole tones from the initial chord [1] [2] [3] [4] [5] [8] [9].

CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: **G**<sub>Δ</sub> **A**<sub>-7</sub> **B**<sub>-7</sub> **C**<sub>Δ</sub> **D**<sub>7</sub> **E**<sub>-7</sub> **F**<sub>♯</sub><sub>♭</sub>

h1 h2 h3 h4 h5 h6 h7

(INVERSE) Tritone Substitutions **5/13**

**F**<sub>♯</sub><sub>♭</sub> **D**<sub>7</sub> | **B**<sub>-7</sub> | **C**<sub>Δ</sub> **B**<sub>-7</sub> **E**<sub>7</sub> **G**<sub>Δ</sub> |





CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\sigma$   
h1 h2 h3 h4 h5 h6 h7

Secondary Dominants INVERSE Substitutions 6/13

F $\sharp$  $\sigma$  D $7$  | B $-7$  C $\Delta$  B $-7$  E $-7$  G $\Delta$  |

CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\sigma$   
h1 h2 h3 h4 h5 h6 h7

(INVERSE) Diatonic Substitutions involving h6 7/13

F $\sharp$  $\sigma$  D $7$  | B $-7$  C $\Delta$  B $-7$  G $\Delta$  |

CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\sigma$   
h1 h2 h3 h4 h5 h6 h7

(INVERSE) Diatonic Substitutions involving h3 8/13

F $\sharp$  $\sigma$  D $7$  | C $\Delta$  D $7$  G $\Delta$  |

CAT CaSe

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\sigma$   
h1 h2 h3 h4 h5 h6 h7

(INVERSE) Diatonic Substitutions involving h7 9/13

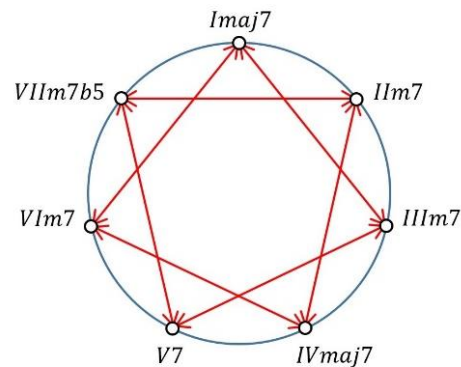
D $7$  | C $\Delta$  D $7$  G $\Delta$  |

## 6. SECONDARY DOMINANTS Inverse Substitutions

*Secondary Dominant Substitution* – Any chord, even if it were to arise from a previous harmonic substitution, can be converted into a Dominant Seventh Chord [1] [2] [3] [4] [5] [8] [9].

## 7. (Inverse) DIATONIC Substitutions involving h6

*Diatonic Substitution* – Two chords that arise from the harmonization of the same scale are interchangeable if the distance between them (between the roots) is equal to a diatonic third (both ascending and descending) [1] [2] [3] [4] [5] [8] [9].



## 8. (Inverse) DIATONIC Substitutions involving h3

\*See point 7 for the definition of *Diatonic Substitution*

## 9. (Inverse) DIATONIC Substitutions involving h7

\*See point 7 for the definition of *Diatonic Substitution*



**CAT CaSe**

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\circ$

h1 h2 h3 h4 h5 h6 h7

(INVERSE) Diatonic Substitutions involving h2 and h4 **10/13**

D $7$  | A $-7$ , D $7$  G $\Delta$  |

10. (Inverse) DIATONIC Substitutions involving  $h_2$  and  $h_4$   
\*See point 7 for the definition of *Diatonic Substitution*

**CAT CaSe**

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\circ$

h1 h2 h3 h4 h5 h6 h7

Contractions (INVERSE Expansion Substitutions) **11/13**

D $7$  | D $7$  G $\Delta$  |

11. CONTRACTION (Inverse EXPANSION)  
*Expansion* – Any Dominant Seventh Chord, by forgoing half of its duration, can be imagined as being preceded by a Minor Seventh or a Half-Diminished Chord distant a descending perfect fourth and characterized by a duration equal to half of the one of the initial chord. In other terms, any Dominant Seventh Chord can be converted into a half-cadence [1] [2] [3] [8] [9].

**CAT CaSe**

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\circ$

h1 h2 h3 h4 h5 h6 h7

Structure Reduction **12/13**

D $7$  | G $\Delta$  |

12. STRUCTURE REDUCTION  
\*See point 2 for the definition of *Structure Reduction*

**CAT CaSe**

**Urge (G major)**  
(CAT CaSe)

G major: G $\Delta$  A $-7$  B $-7$  C $\Delta$  D $7$  E $-7$  F $\sharp$  $\circ$

h1 h2 h3 h4 h5 h6 h7

SHOW INVERSE PROCEDURE **Final Outcome** **13/13**

D $7$	D $7$	D $7$	D $7$
E $\Delta$	D $\flat$ $\Delta$	D $\Delta$	B $\Delta$
D $7$	D $7$	G $\Delta$	G $\Delta$
C $\Delta$	A $\Delta$	B $\flat$ $7$	G $\Delta$

13. FINAL OUTCOME



### Inverse Procedure

CAT CaSe	CAT CaSe	CAT CaSe																								
<b>Urge (G major)</b> (CAT CaSe)	<b>Urge (G major)</b> (CAT CaSe)	<b>Urge (G major)</b> (CAT CaSe)																								
<b>1/9</b> Inverse Procedure - Initial Progression	<b>2/9</b> Expansion	<b>3/9</b> Diatonic Substitutions																								
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CAT CaSe	CAT CaSe	CAT CaSe																								
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$E_\Delta$	$D^\flat_\Delta$	$D_\Delta$	$B_\Delta$																							
$C_\Delta$	$A_\Delta$	$B^\flat_7$	$G_\Delta$																							





**Example #2 (Minor Key):** “Lullaby of Birdland” (George Shearing) – first 8 bars

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sub>-7</sub> G<sub>ø</sub> A<sub>Δ</sub><sup>b</sup> B<sub>-7</sub><sup>b</sup> C<sub>-7</sub> D<sub>Δ</sub><sup>b</sup> E<sub>7</sub><sup>b</sup>

h1 h2 h3 h4 h5 h6 h7

Initial Progression **1/12**

F<sub>-7</sub> D<sub>ø</sub> | G<sub>7</sub> C<sub>7</sub> | F<sub>-7</sub> | B<sub>-7</sub><sup>b</sup> E<sub>7</sub><sup>b</sup> |

A<sub>Δ</sub><sup>b</sup> F<sub>-7</sub> | B<sub>-7</sub><sup>b</sup> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> | G<sub>ø</sub> C<sub>7</sub> |

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sub>-7</sub> G<sub>ø</sub> A<sub>Δ</sub><sup>b</sup> B<sub>-7</sub><sup>b</sup> C<sub>-7</sub> D<sub>Δ</sub><sup>b</sup> E<sub>7</sub><sup>b</sup>

h1 h2 h3 h4 h5 h6 h7

Structure Reduction **2/12**

F<sub>-7</sub> D<sub>ø</sub> G<sub>7</sub> C<sub>7</sub> | F<sub>-7</sub> B<sub>-7</sub><sup>b</sup> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> F<sub>-7</sub> B<sub>-7</sub><sup>b</sup> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> G<sub>ø</sub> C<sub>7</sub> |

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sub>-7</sub> G<sub>ø</sub> A<sub>Δ</sub><sup>b</sup> B<sub>-7</sub><sup>b</sup> C<sub>-7</sub> D<sub>Δ</sub><sup>b</sup> E<sub>7</sub><sup>b</sup>

h1 h2 h3 h4 h5 h6 h7

Contractions (INVERSE Expansion Substitutions) **3/12**

F<sub>-7</sub> D<sub>ø</sub> G<sub>7</sub> C<sub>7</sub> | F<sub>-7</sub> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> F<sub>-7</sub> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> C<sub>7</sub> |

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sub>-7</sub> G<sub>ø</sub> A<sub>Δ</sub><sup>b</sup> B<sub>-7</sub><sup>b</sup> C<sub>-7</sub> D<sub>Δ</sub><sup>b</sup> E<sub>7</sub><sup>b</sup>

h1 h2 h3 h4 h5 h6 h7

INVERSE Substitutions (Modal Interchange) involving Half Diminished Chords different from h2 **4/12**

F<sub>-7</sub> D<sub>Δ</sub><sup>b</sup> G<sub>7</sub> C<sub>7</sub> | F<sub>-7</sub> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> F<sub>-7</sub> E<sub>7</sub><sup>b</sup> | A<sub>Δ</sub><sup>b</sup> C<sub>7</sub> |

**1. INITIAL STRUCTURE and HARMONIZATION VECTOR**

*Harmonization Vector* – The vector whose components represent the seventh chords that arise from the harmonization of the scale (in this case, the Aeolian of F) [10] [11].

**2. STRUCTURE REDUCTION**

*Structure Reduction* – Without “Structure Reduction” a correct application of CAT (*Cataldo Advanced Transformations*) is de facto impossible. Very simply, the number of bars and the duration of the chords are iteratively halved. The procedure is stopped the moment in which even a single chord characterized by a duration equal to a beat appears. “Structure Reduction” is applied every time it is possible, so as to obtain the highest simplification level [1] [8] [9].

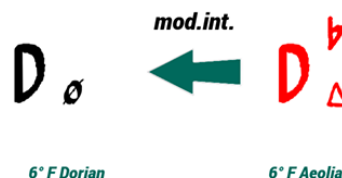
**3. CONTRACTION (Inverse EXPANSION)**

*Expansion* – Any Dominant Seventh Chord, by forgoing half of its duration, can be imagined as being preceded by a Minor Seventh or a Half-Diminished Chord distant a descending perfect fourth and characterized by a duration equal to half of the one of the initial chord. In other terms, any Dominant Seventh Chord can be converted into a half-cadence [1] [2] [3] [8] [9].

**4. Inverse Substitutions (Modal Interchange) involving HALF – DIMINISHED CHORDS different from h2**

*Modal Interchange* – Two chords that arise from the harmonization of two different scales characterized by the same tonic (generic parallel keys) are interchangeable if they are placed in the same position (if they represent the same harmonic degree) [1] [4] [5].

**Bar Substitutions**





**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>o</sup> A<sup>b</sup><sub>Δ</sub> B<sup>b</sup><sub>-7</sub> C<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> E<sup>b</sup><sub>7</sub>

h1 h2 h3 h4 h5 h6 h7

Secondary Dominants *INVERSE* Substitutions **5/12**

| F<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> G<sup>o</sup> C<sup>7</sup> | F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> C<sup>7</sup> |

### 5. SECONDARY DOMINANTS Inverse Substitutions

*Secondary Dominant Substitution* – Any chord, even if it were to arise from a previous harmonic substitution, can be converted into a Dominant Seventh Chord [1] [2] [3] [4] [5] [8] [9].

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>o</sup> A<sup>b</sup><sub>Δ</sub> B<sup>b</sup><sub>-7</sub> C<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> E<sup>b</sup><sub>7</sub>

h1 h2 h3 h4 h5 h6 h7

Contractions (*INVERSE* Expansion Substitutions) **6/12**

| F<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> C<sup>7</sup> | F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> C<sup>7</sup> |

### 6. CONTRACTION (Inverse EXPANSION)

\*See point 3 for the definition of *Contraction*

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>o</sup> A<sup>b</sup><sub>Δ</sub> B<sup>b</sup><sub>-7</sub> C<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> E<sup>b</sup><sub>7</sub>

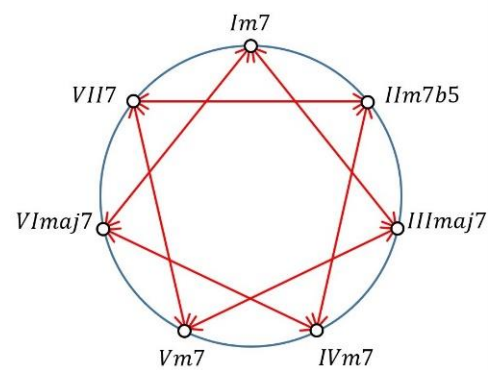
h1 h2 h3 h4 h5 h6 h7

(*INVERSE*) Diatonic Substitutions involving h6 **7/12**

| F<sup>-7</sup> C<sup>7</sup> | F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | A<sup>b</sup><sub>Δ</sub> C<sup>7</sup> |

### 7. (Inverse) DIATONIC Substitutions involving h<sub>6</sub>

*Diatonic Substitution* – Two chords that arise from the harmonization of the same scale are interchangeable if the distance between them (between the roots) is equal to a diatonic third (both ascending and descending [1] [2] [3] [4] [5] [8] [9]).



**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>o</sup> A<sup>b</sup><sub>Δ</sub> B<sup>b</sup><sub>-7</sub> C<sup>-7</sup> D<sup>b</sup><sub>Δ</sub> E<sup>b</sup><sub>7</sub>

h1 h2 h3 h4 h5 h6 h7

(*INVERSE*) Diatonic Substitutions involving h3 **8/12**

| F<sup>-7</sup> C<sup>7</sup> | F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | F<sup>-7</sup> E<sup>b</sup><sub>7</sub> | F<sup>-7</sup> C<sup>7</sup> |

### 8. (Inverse) DIATONIC Substitutions involving h<sub>3</sub>

\*See point 7 for the definition of *Diatonic Substitution*



**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>ø</sup> A<sup>Δ</sup> B<sup>b7</sup> C<sup>-7</sup> D<sup>Δ</sup> E<sup>b7</sup>  
h1 h2 h3 h4 h5 h6 h7

(INVERSE) Diatonic Substitutions involving h7 **9/12**

| F<sup>-7</sup> C<sup>7</sup> | F<sup>-7</sup> C<sup>-7</sup> | F<sup>-7</sup> C<sup>-7</sup> | F<sup>-7</sup> C<sup>7</sup> |

9. (Inverse) DIATONIC Substitutions involving h7

\*See point 7 for the definition of *Diatonic Substitution*

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>ø</sup> A<sup>Δ</sup> B<sup>b7</sup> C<sup>-7</sup> D<sup>Δ</sup> E<sup>b7</sup>  
h1 h2 h3 h4 h5 h6 h7

Structure Reduction **10/12**

| F<sup>-7</sup> C<sup>7</sup> F<sup>-7</sup> C<sup>-7</sup> | F<sup>-7</sup> C<sup>-7</sup> F<sup>-7</sup> C<sup>7</sup> |

10. STRUCTURE REDUCTION

\*See point 2 for the definition of *Structure Reduction*

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>ø</sup> A<sup>Δ</sup> B<sup>b7</sup> C<sup>-7</sup> D<sup>Δ</sup> E<sup>b7</sup>  
h1 h2 h3 h4 h5 h6 h7

Further Secondary Dominants INVERSE Substitutions **11/12**

| F<sup>-7</sup> C<sup>-7</sup> F<sup>-7</sup> C<sup>-7</sup> | F<sup>-7</sup> C<sup>-7</sup> F<sup>-7</sup> C<sup>-7</sup> |

11. Further SECONDARY DOMINANT Inverse Substitutions

\*See point 5 for the definition of *Secondary Dom. Substitutions*

**CAT CaSe**

**Lullaby (F minor)**  
(CAT CaSe)

F minor: F<sup>-7</sup> G<sup>ø</sup> A<sup>Δ</sup> B<sup>b7</sup> C<sup>-7</sup> D<sup>Δ</sup> E<sup>b7</sup>  
h1 h2 h3 h4 h5 h6 h7

SHOW INVERSE PROCEDURE **Final Outcome** **12/12**

| F<sup>-7</sup> D<sup>ø</sup> | G<sup>7</sup> C<sup>7</sup> | F<sup>-7</sup> | B<sup>b7</sup> E<sup>b7</sup> |

| A<sup>Δ</sup> F<sup>-7</sup> | B<sup>b7</sup> E<sup>b7</sup> | A<sup>Δ</sup> | G<sup>ø</sup> C<sup>7</sup> |

12. FINAL OUTCOME



### Inverse Procedure

CAT CaSe	CAT CaSe	CAT CaSe
<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)
F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>
<b>Inverse Procedure - Initial Progression</b>	<b>Secondary Dominants Substitutions</b>	<b>Diatonic Substitutions</b>
<b>1/9</b>	<b>2/9</b>	<b>3/9</b>
I            V            I            V   F <sub>-7</sub>   C <sub>-7</sub>   F <sub>-7</sub>   C <sub>-7</sub>   I            V            I            V   F <sub>-7</sub>   C <sub>-7</sub>   F <sub>-7</sub>   C <sub>-7</sub>	F <sub>-7</sub>   C <sub>7</sub>   F <sub>-7</sub>   C <sub>-7</sub>     F <sub>-7</sub>   C <sub>-7</sub>   F <sub>-7</sub>   C <sub>7</sub>	F <sub>-7</sub>   C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   F <sub>-7</sub>   C <sub>7</sub>

CAT CaSe	CAT CaSe	CAT CaSe
<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)
F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>
<b>Diatonic Substitutions</b>	<b>Diatonic Substitutions</b>	<b>Expansion</b>
<b>4/9</b>	<b>5/9</b>	<b>6/9</b>
F <sub>-7</sub>   C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   C <sub>7</sub>	F <sub>-7</sub>   D <sub>Δ</sub> <sup>b</sup> C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   C <sub>7</sub>	F <sub>-7</sub>   D <sub>Δ</sub> <sup>b</sup> G <sub>o</sub> C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   C <sub>7</sub>

CAT CaSe	CAT CaSe	CAT CaSe
<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)	<b>Lullaby (F minor)</b> (CAT CaSe)
F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>	F minor: F <sub>-7</sub> G <sub>o</sub> A <sub>Δ</sub> <sup>b</sup> B <sub>-7</sub> <sup>b</sup> C <sub>-7</sub> D <sub>Δ</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup> <small>h1 h2 h3 h4 h5 h6 h7</small>
<b>Secondary Dominants Substitutions</b>	<b>Modal Interchange and/or Similitude</b>	<b>Expansion</b>
<b>7/9</b>	<b>8/9</b>	<b>9/9</b>
F <sub>-7</sub>   D <sub>Δ</sub> <sup>b</sup> G <sub>7</sub> C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   C <sub>7</sub>	F <sub>-7</sub>   D <sub>o</sub> G <sub>7</sub> C <sub>7</sub>   F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   C <sub>7</sub>	F <sub>-7</sub>   D <sub>o</sub> G <sub>7</sub> C <sub>7</sub>   F <sub>-7</sub>   B <sub>-7</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup>     A <sub>Δ</sub> <sup>b</sup> F <sub>-7</sub>   B <sub>-7</sub> <sup>b</sup> E <sub>7</sub> <sup>b</sup>   A <sub>Δ</sub> <sup>b</sup>   G <sub>o</sub> C <sub>7</sub>

### Remarks

It's worth underlining how the chord progression analysis has almost nothing to do with the improvisation built on the original structure. More precisely, net of a certain "Horizontalization" the musician can exploit in facing specific harmonic aggregates, such as "Tonicizations" and "Turnarounds", the improvisation should be carried out "vertically", abiding by the local harmony.

The Local Tonal Centre can be characterized by significant fluctuations: sometimes, it is very difficult to correctly identify it, in particular when a progression cannot be regarded as manifestly "tonal" (built on cadences). And especially in this case, paradoxically, the musician should improvise by abiding by the harmonic progression, chord by chord [12 – 29].





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